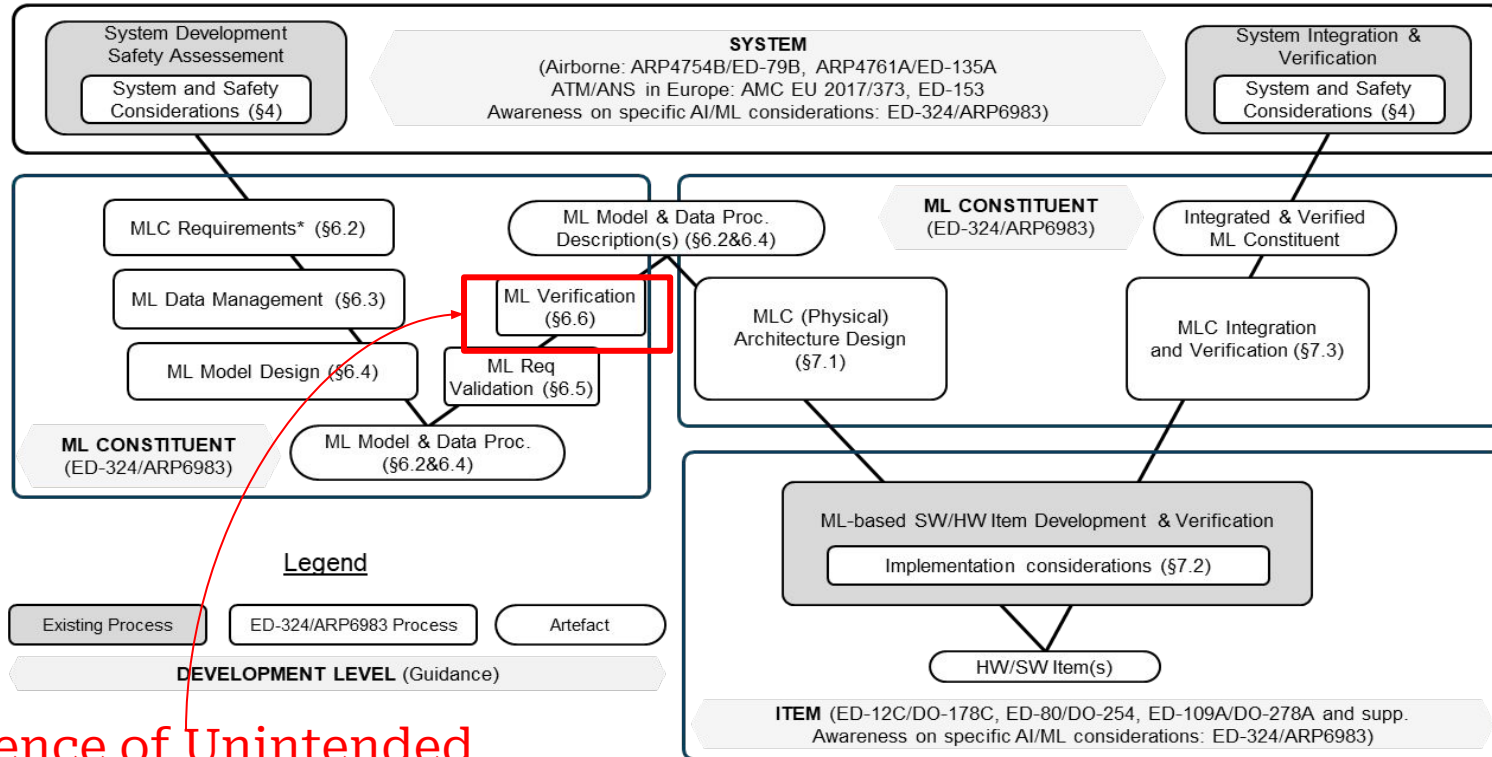

Robustness Verification: Neural Network's Surrogate

Melanie Ducoffe

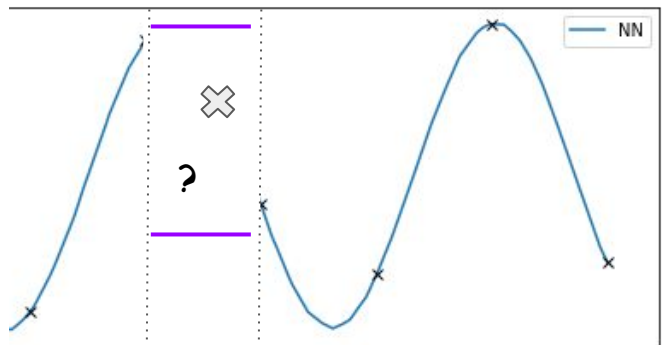
ANITI DAYS

November 2024

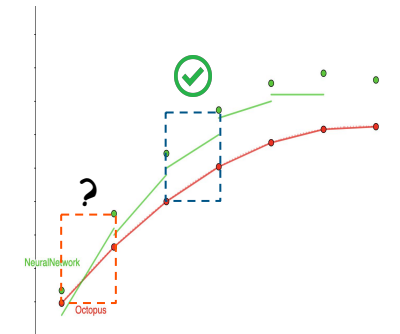
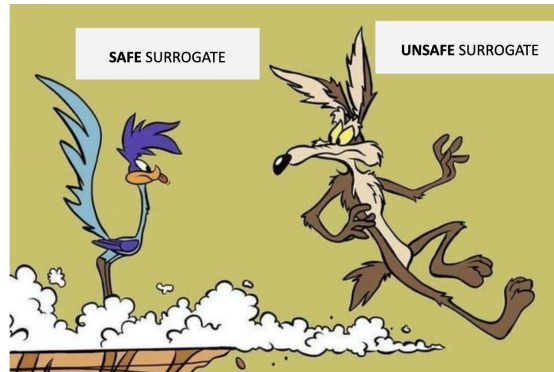




Absence of Unintended
Functionality



$$\max_{x \in \Omega} |f(x) - f(x_0)|$$



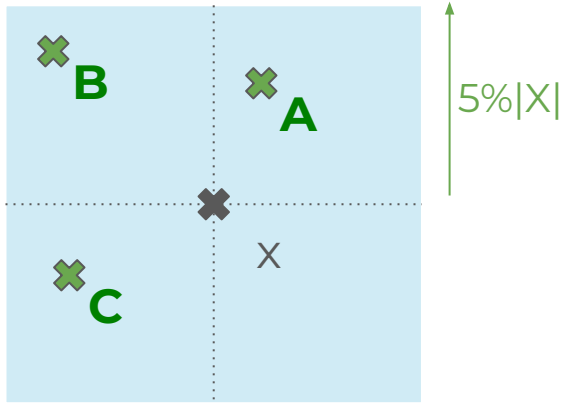
$$\min_{x \in \Omega} f(x) - f(x_0) \geq 0$$

Partial Input Monotony

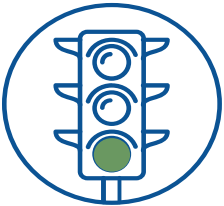
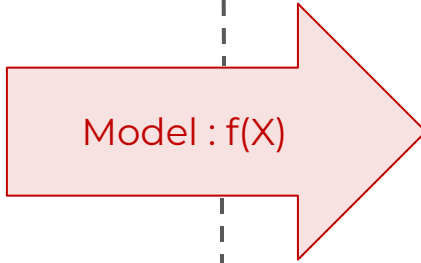
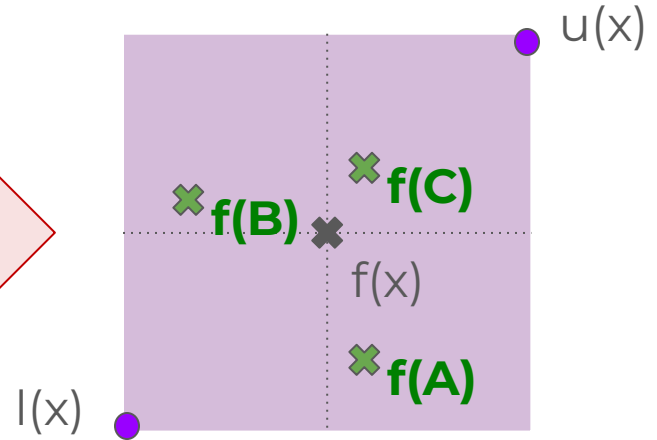


x_1	x_2	$f(x_1)$	$f(x_2)$
$\begin{pmatrix} \text{speed} \\ \text{weight} \\ \text{dry runway} \end{pmatrix}$	$\begin{pmatrix} \text{speed} \\ \text{weight} \\ \text{wet runway} \end{pmatrix}$		
		\implies	$\text{BDE}_1 < \text{BDE}_2$

INPUT DOMAIN Ω



OUTPUT DOMAIN

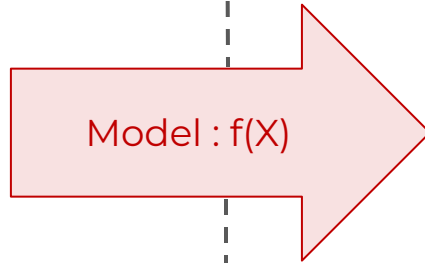
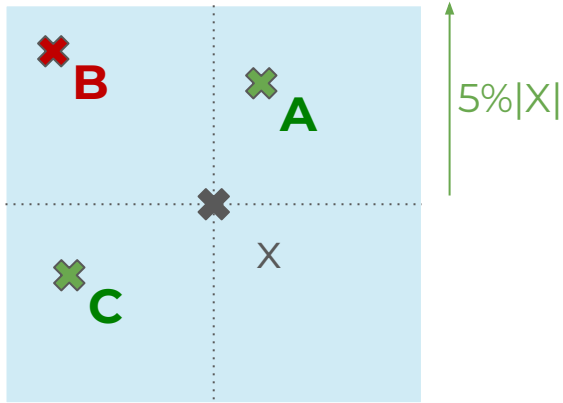


The property is **verified** !

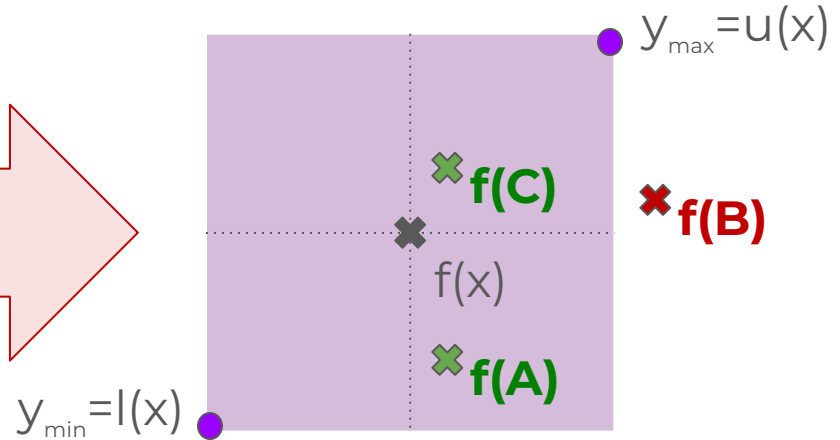
$$\max_{z \in \Omega} g(Z;X) \leq 0$$

$$g(Z;X) = \max_i \max(f_i(Z) - u_i(X), l_i(X) - f_i(Z))$$

INPUT DOMAIN Ω



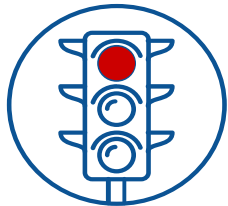
OUTPUT DOMAIN

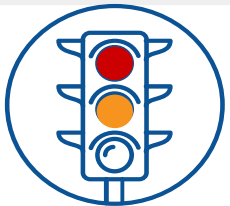


$$\max_{z \in \Omega} g(z; X) > 0$$

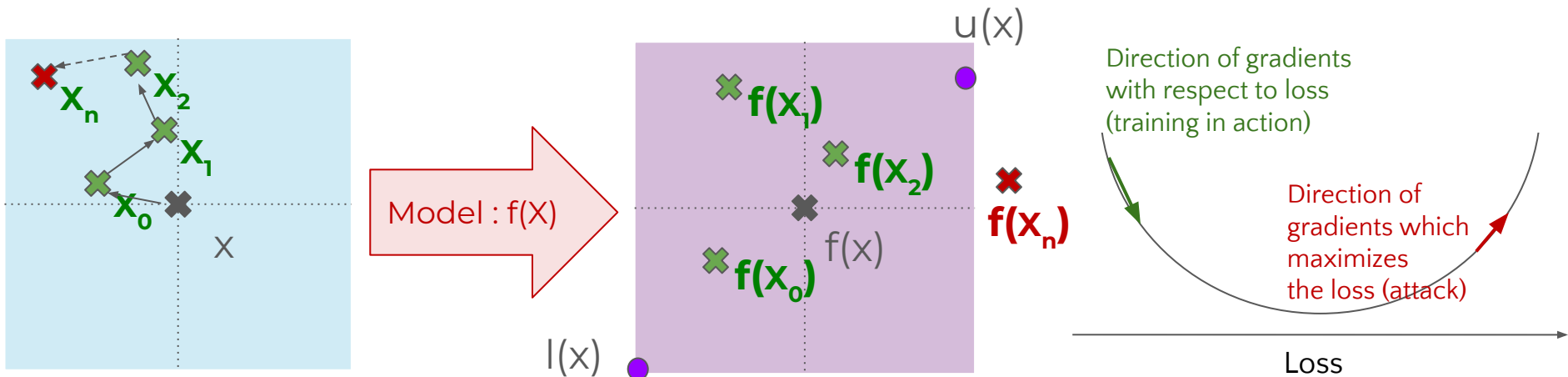
$$g(z; X) = \max_i \max(f_i(z) - u_i(X), l_i(X) - f_i(z))$$

The property is **violated**



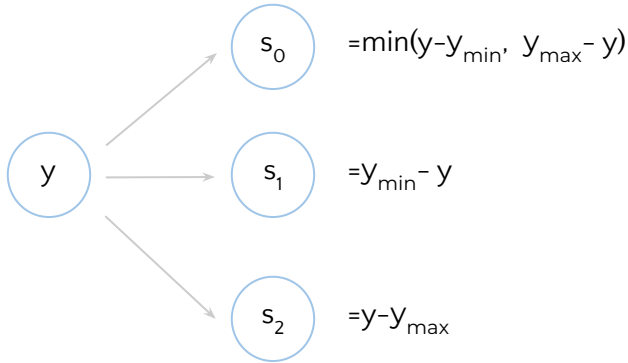


$$\max_{Z \in \Omega} g(Z; X) > 0 \Rightarrow \exists Z \in \Omega \text{ s.t } g(Z; X) > 0$$



Shows model vulnerabilities but not their absence
 >> Do not provide property verification guarantee.

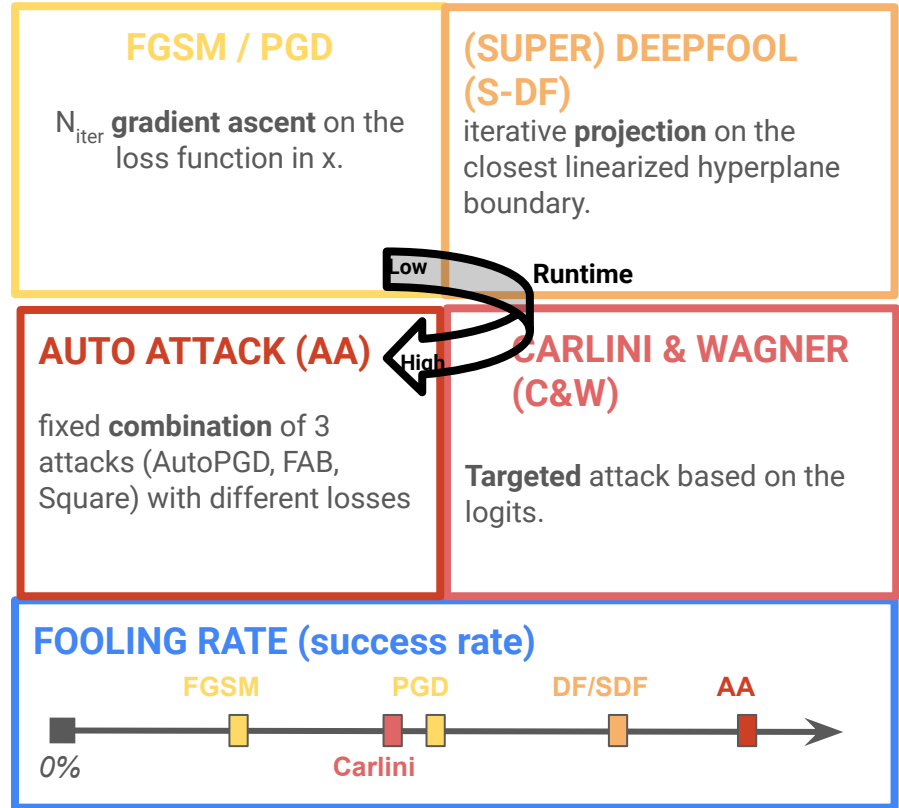
Naïve attacks schemes can be used for regression (FGSM, PGD)

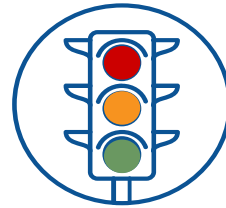
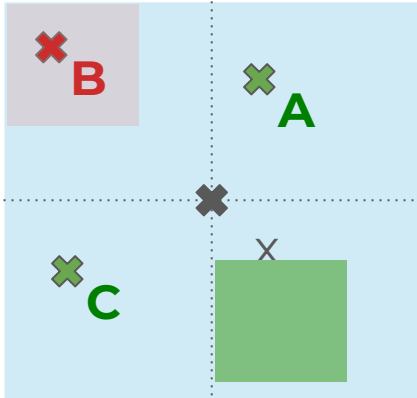


$$\max_{z \in \Omega} g(z; X) \leq 0$$



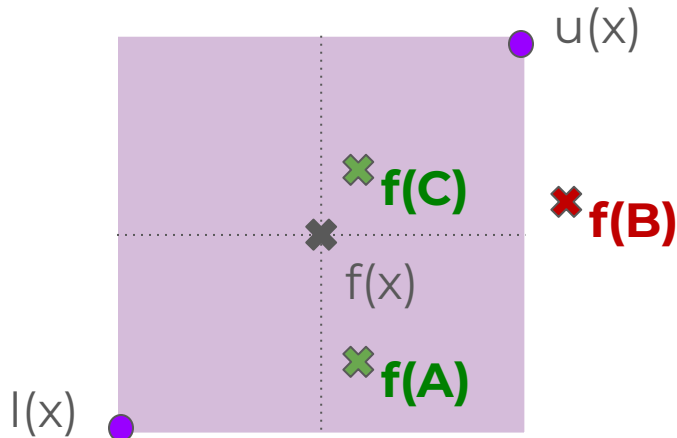
$$\operatorname{argmax}_{z \in \Omega} s(z; X) = 0$$





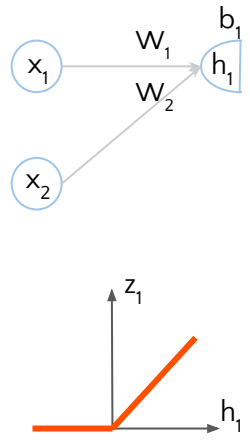
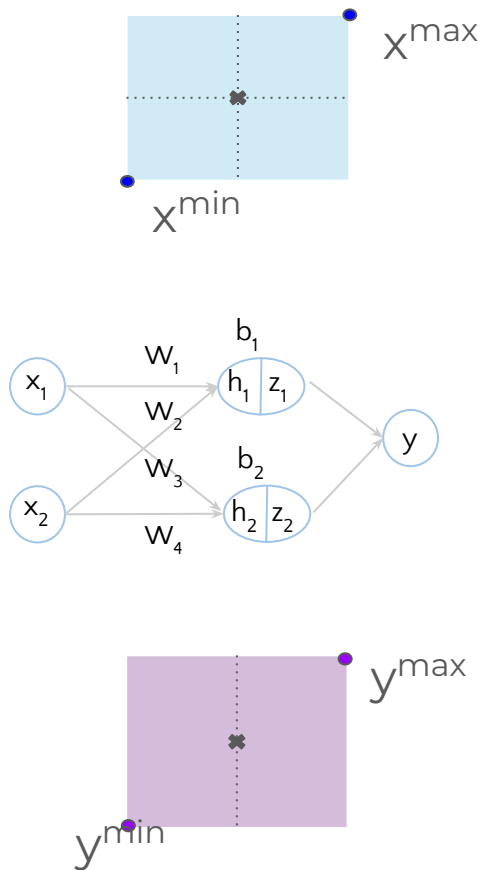
$$\max_{z \in \Omega} g(z; X) > 0$$

- + convergence to the true optimum that implies robustness or non robustness.
- + Not scalable to larger network
SMT-solver [Marabou]
Lipschitz optimization (Paul Novello)
Mixed Integer Programming (VENUS)



No magical trick:
white box setting





Bounding the input perturbation

$$x_i^{\min} \leq x_i \leq x_i^{\max}$$

Encoding Neural Network

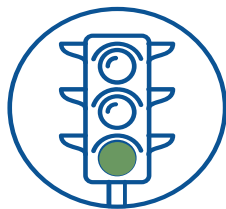
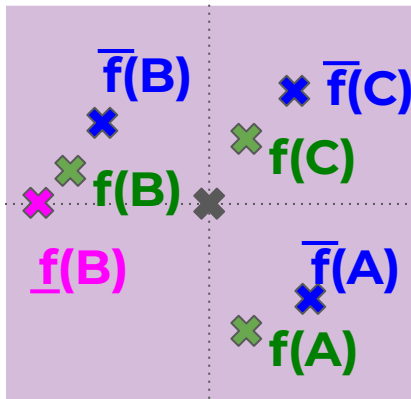
$$\begin{aligned} h_1 &= w_1 \cdot x_1 + w_2 \cdot x_2 + b_1 \\ h_2 &= w_3 \cdot x_1 + w_4 \cdot x_2 + b_2 \end{aligned}$$

Big M encoding for ReLU

$$\begin{aligned} z_1 &= \max(0, h_1) \\ z_2 &= \max(0, h_2) \end{aligned} \equiv \begin{cases} z_i \geq 0 \dots \\ z_i \geq h_i \\ \delta_i \in \{0, 1\}^{\text{ReLU}} \end{cases} \quad \begin{cases} z_i \leq u_i \cdot \delta_i \\ z_i \leq h_i - (1 - \delta_i) \end{cases}$$

Encoding property violation (SAT)

$$y_1^{\max} < y_1 \text{ or } y_1 < y_1^{\min} \text{ etc.}$$



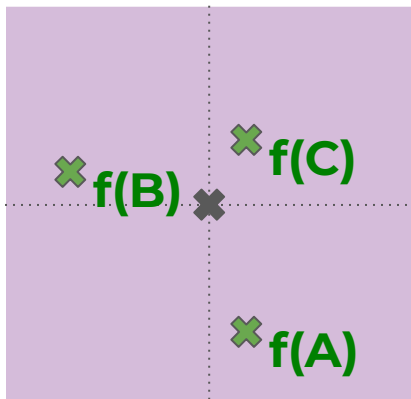
Build Under/Over approximation of f

$$\forall z \in \underline{\Omega} \quad \underline{f}(z) \leq f(z) \leq \bar{f}(z)$$

Use it for dominating g

$$\forall z \in \Omega \quad \bar{g}(z) \leq g(z)$$

$$\max_{z \in \Omega} \bar{g}(z; X) \leq 0 \Rightarrow \max_{z \in \Omega} g(z; X) \leq 0$$



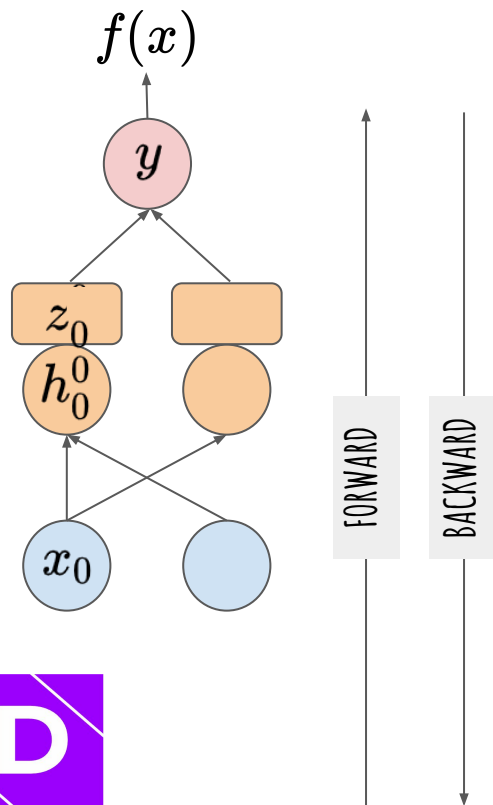
$\times \bar{f}(C)$



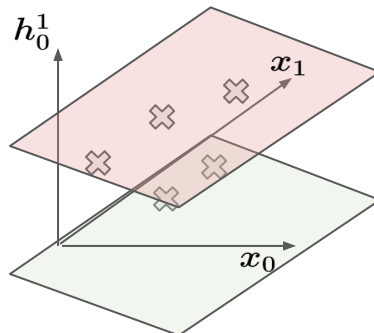
outer-approximations that only implies robustness:

Linear Relaxation [CROWN]
Convex Relaxation [SDP]



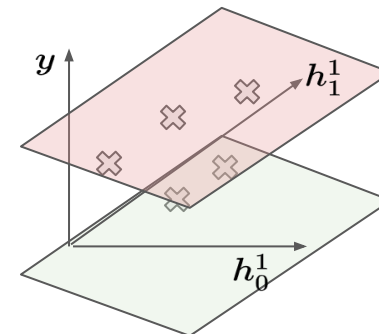


VERIFICATION 'FORWARD-FEED'



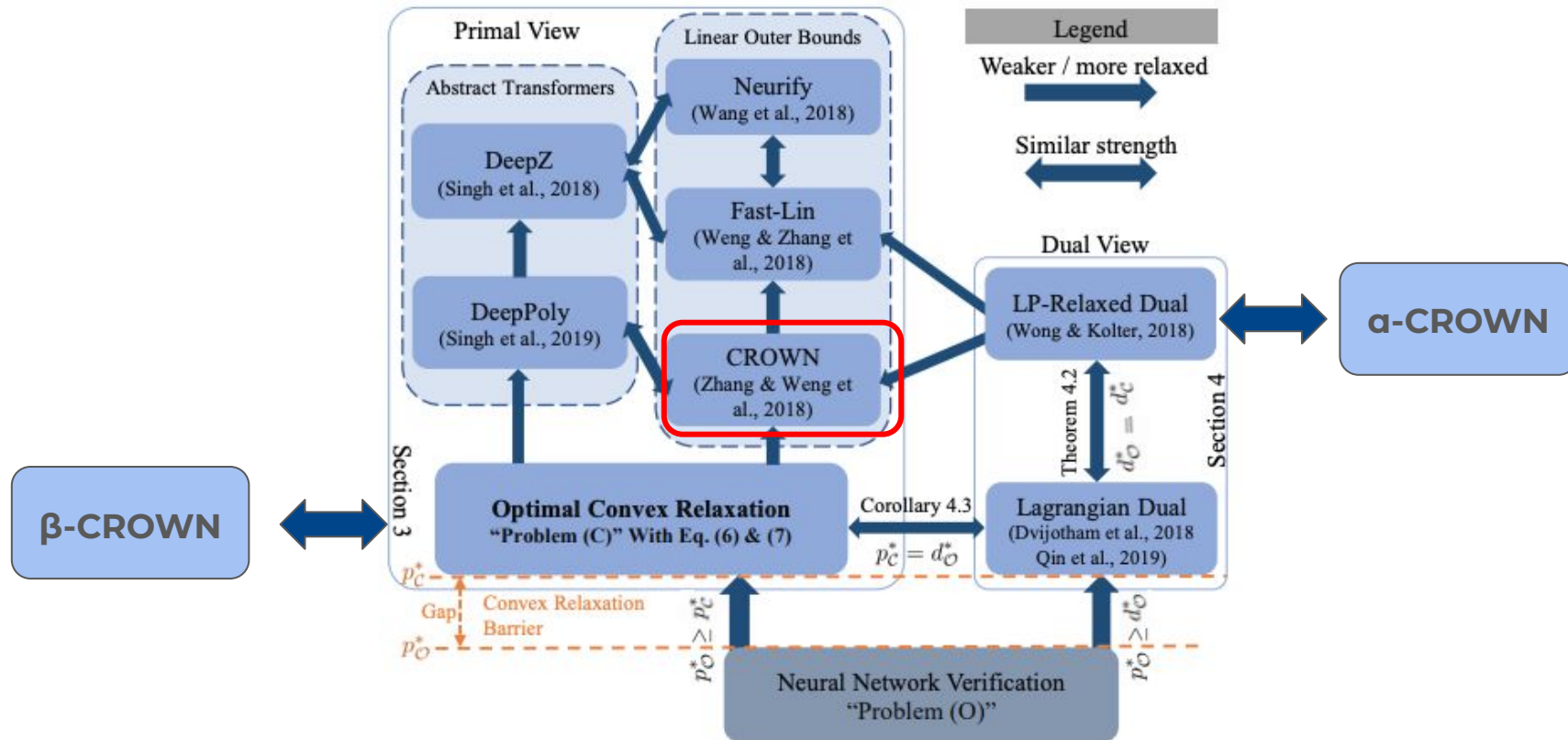
- + Self-sufficient
- + Complexity = cost of inference
- Less accurate
- Not scalable on large images

VERIFICATION 'BACKWARD-FEED'



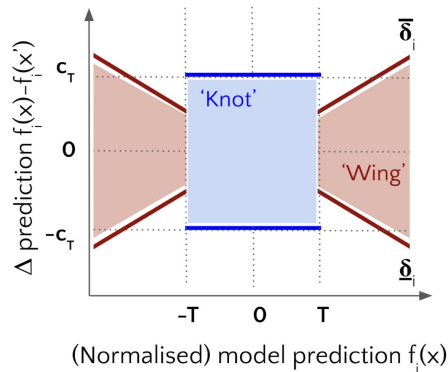
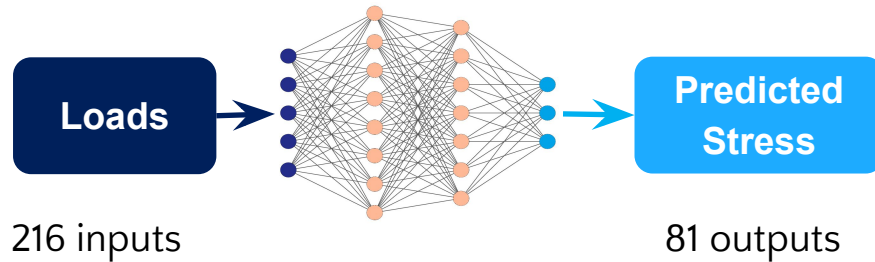
- + Not self-sufficient (pre-processing)
- + Complexity = cost of backpropagation at best (gradient)
- More accurate
- Not scalable on large outputs (GAN)





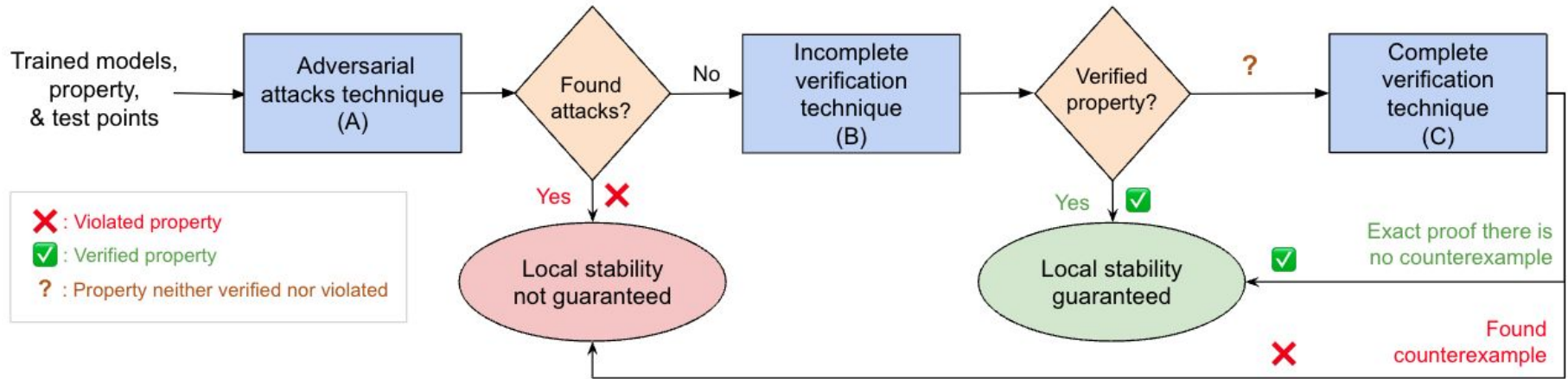
A Convex Relaxation Barrier to Tigh Robustness Verification of NNs, Salman et al.

Aircraft Loads-to-Stress Prediction

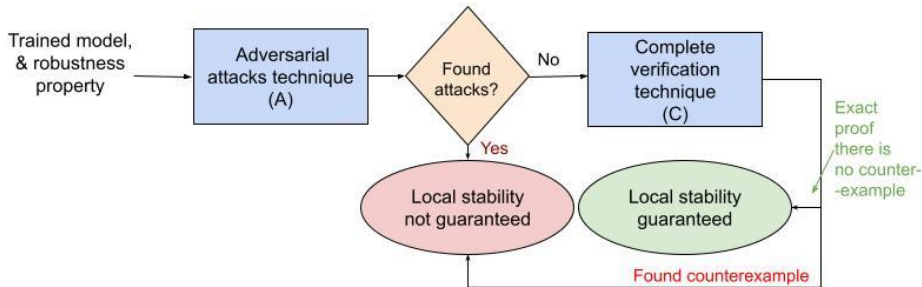


- Model- Research prototypes:
 - Two hidden layers (165 neurons)
 - ReLu activation functions
 - Dense output layer (81)
- Test data: 1000 loads/stress points

Verification approach - Combinaison



A+B+C



A+C

- A: PGD (Cleverhans)
- B: CROWN (Decomon)
- C: MILP (Gurobi)

Results

	(1) A	(2) B	(3) C	(4) A+C	(5) B+C	(6) Pipeline A+B+C
#Tested	1000	1000	1000	1000/558	1000/446	1000/558/4
#True	-	554	558	558	558	-/554/4 = 558
#False	442	-	442	442	442	442/-/0 = 442
Runtime	10.7	3.3	267	19.8	267	10.7/1.96/3.91 = 16.6

-45% of test data are shown to be non locally stable

The “Adversarial attacks” step was able to find all non-stabilities

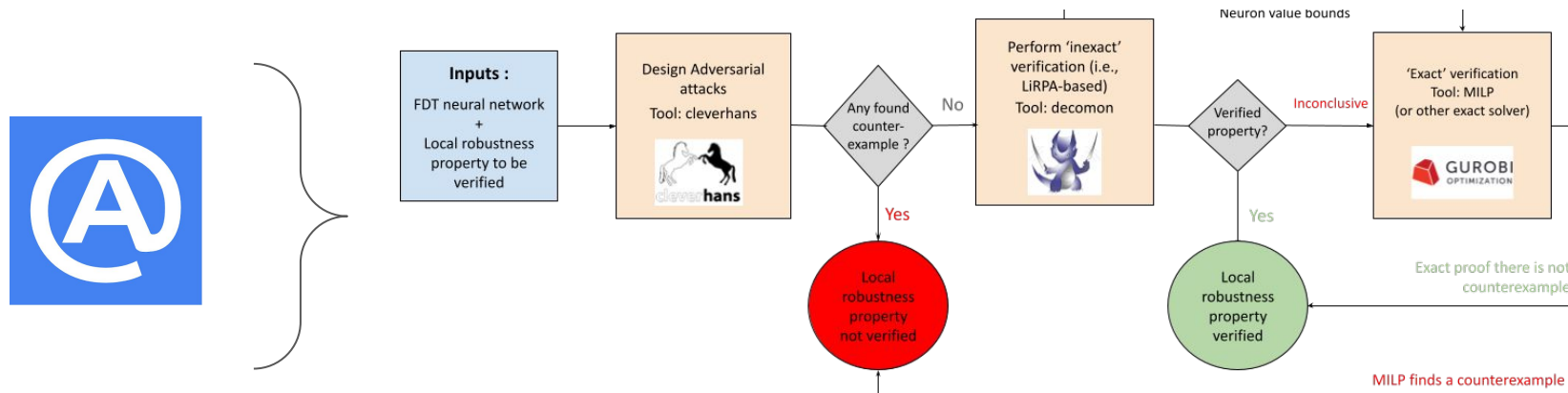
Low number of remaining test data to be evaluated by “C” after (A or A+B)

Significant decrease in computational time

Open source library: Airobas



Stability accuracy can be efficiently measured with a verification pipeline



Current models have **deceiving stability accuracy: -40%**. What tools are at our disposal ?

- 1) XAI actionability: Reducing the problem complexity (input and output dimensions)
- 2) Regularizing the training to balance between good regression performance and good stability accuracy

Robust  Accurate  Stable

Data Augmentation

Artificially increase the size of the dataset by applying domain-specific transformations on the input and output data. It introduces stability invariance.

Weights Constraints

Weights constraints limit the Lipschitz constant in a neural network. It is known to increase the model's resilience against adversarial attacks or input perturbations by limiting the model's capacity to fit noise

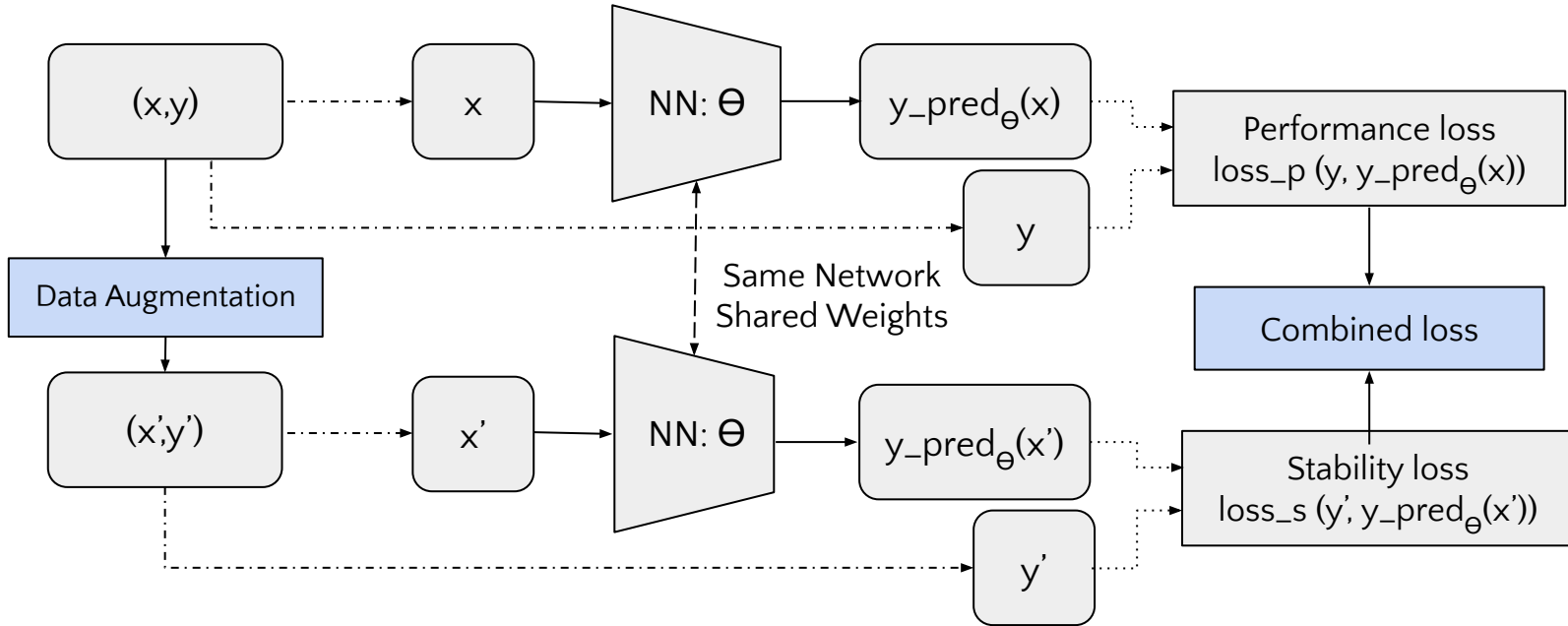
Certified Training (Meta Networks)

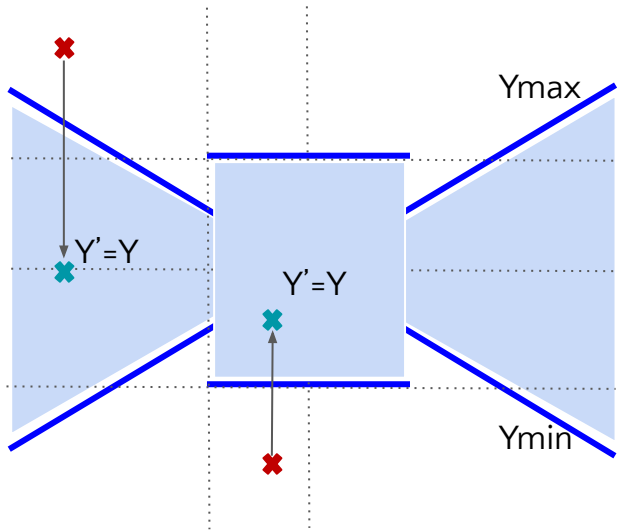
Certified training use Incomplete Formal Methods as a Meta Model to provide formal guarantees about a model's robustness against domain-specific perturbations.

Data Augmentation

Artificially increase the size of the dataset by applying domain-specific transformations on the input and output data. It introduces stability invariance.

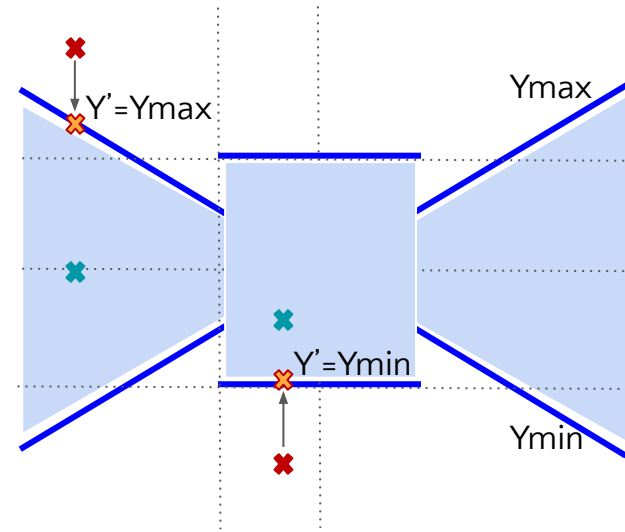
Enhance stability during design/training via targeted data augmentation





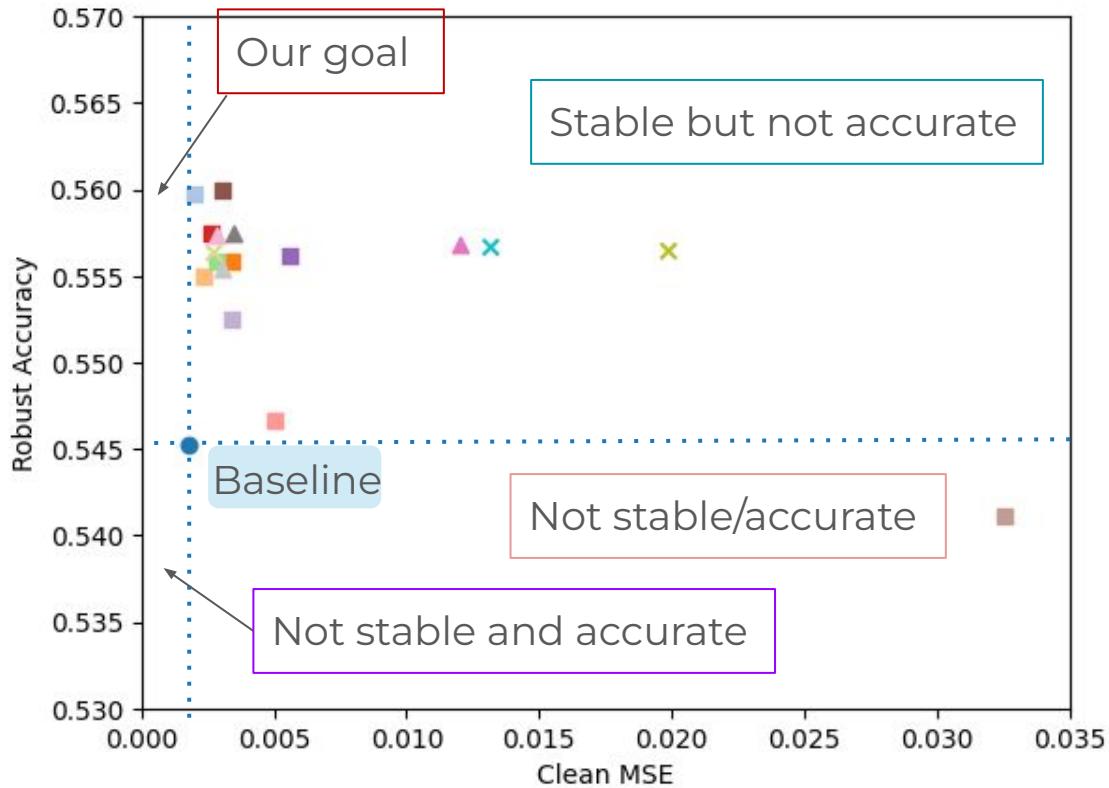
Groundtruth

use the groundtruth label of the initial input



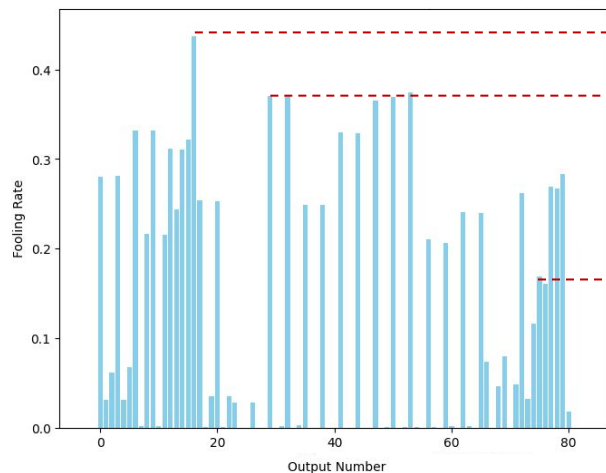
Stability clipping

clip the prediction of the adversarial input to lie within the stability bounds ($Ymin, Ymax$)

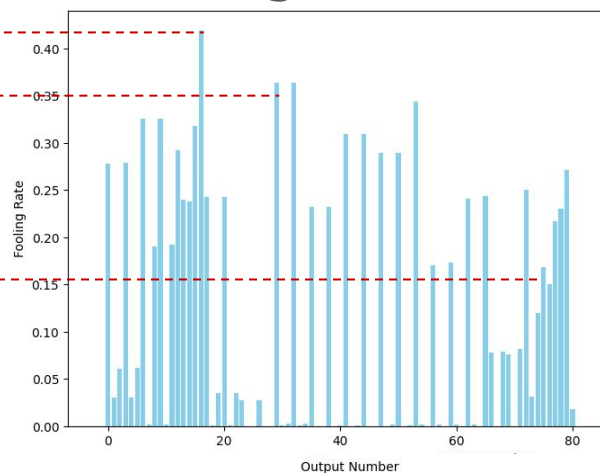


$$\text{Fooling Rate} = 1 - \text{Robust_accuracy}$$

Baseline



Data Augmentation

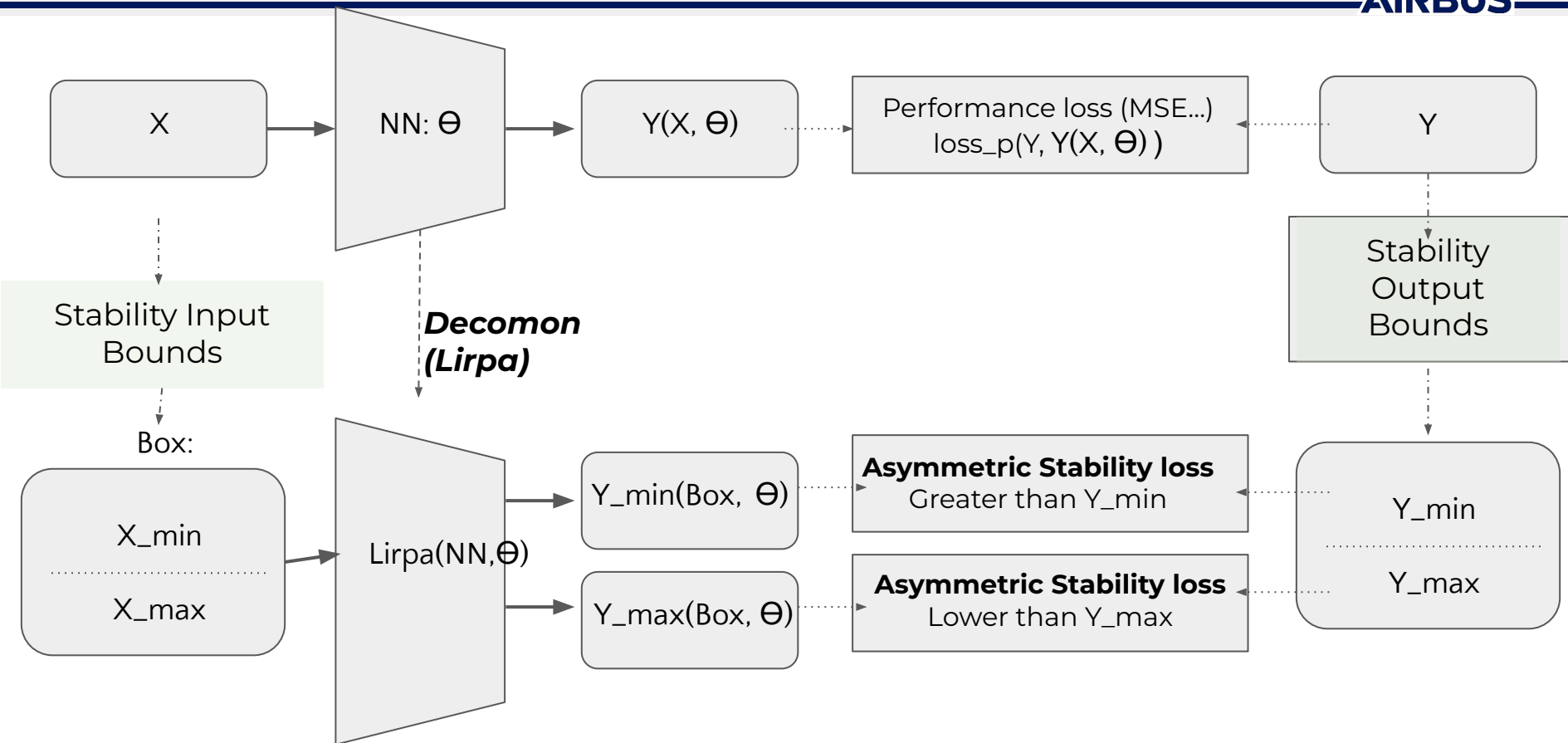


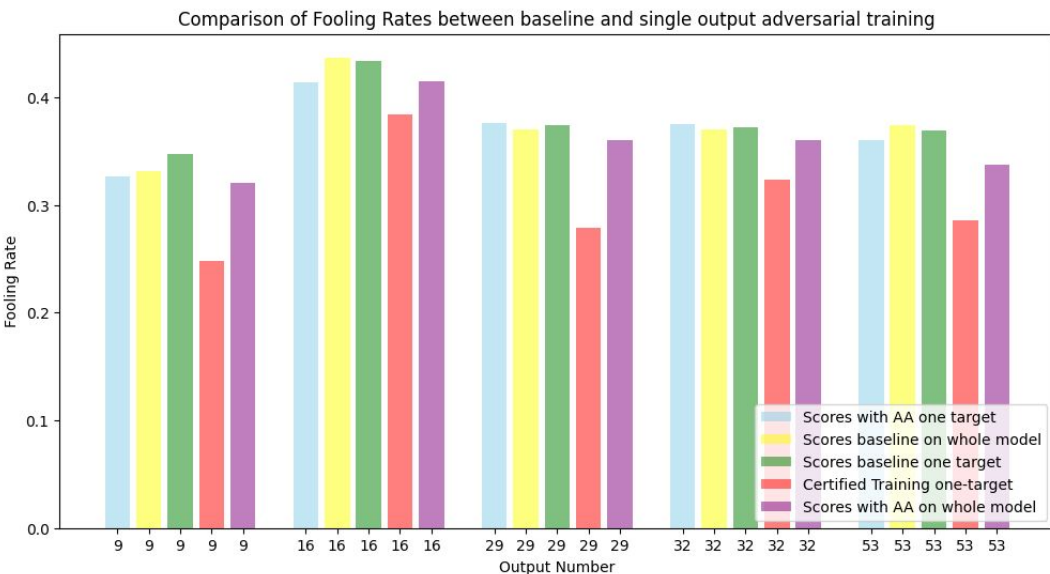
- Most of the outputs are **naturally Robust**
- About **40%** of outputs are **problematic**

→ Can we **target** those outputs ?

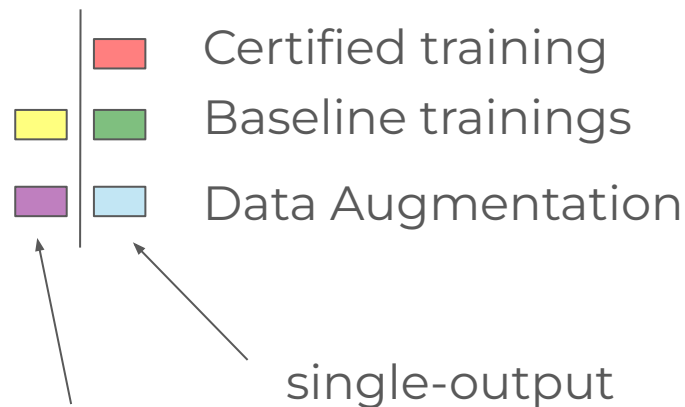
Certified Training (Meta Networks)

Certified training use Incomplete Formal Methods as a Meta Model to provide formal guarantees about a model's robustness against domain-specific perturbations.





- **CT for single-output models** for the 5 problematic outputs



single-output

multi-output

Promising results ! 5-10% drop in the fooling rates compared to the previous models

Surrogate Neural Networks Local Stability for Aircraft Predictive Maintenance, FMICS 2024



Thomas Deltort



Ryma Boumazouza



Guillaume Poveda



Marion Cécile Martin



Audrey Galametz

ANITI EVENT: Hands on Verification
6th March 2025

